Formulating A Self-Adaptive Dynamic Tolling Strategy for High Occupancy Toll (HOT) Lane Operations

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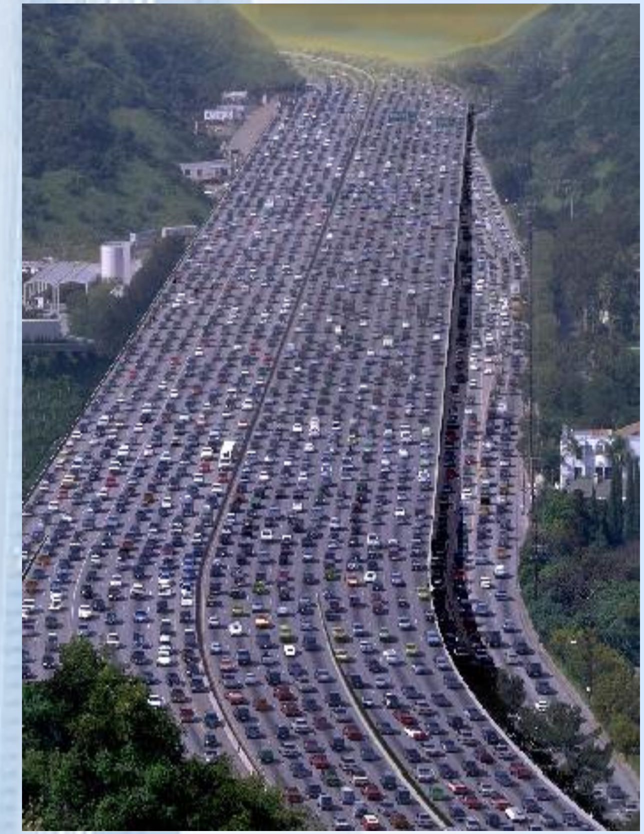
Traffic Condition Is Getting Worse



Source: http://home.fuse.net

Traffic Congestion Costs

- In the 85-surveyed-urban areas
  - Annual person delay: 47 hours
  - Average of $794 per traveler

2005 Urban Mobility Report

Source: http://ops.fhwa.dot.gov

High Occupancy Vehicle (HOV) Lane Systems

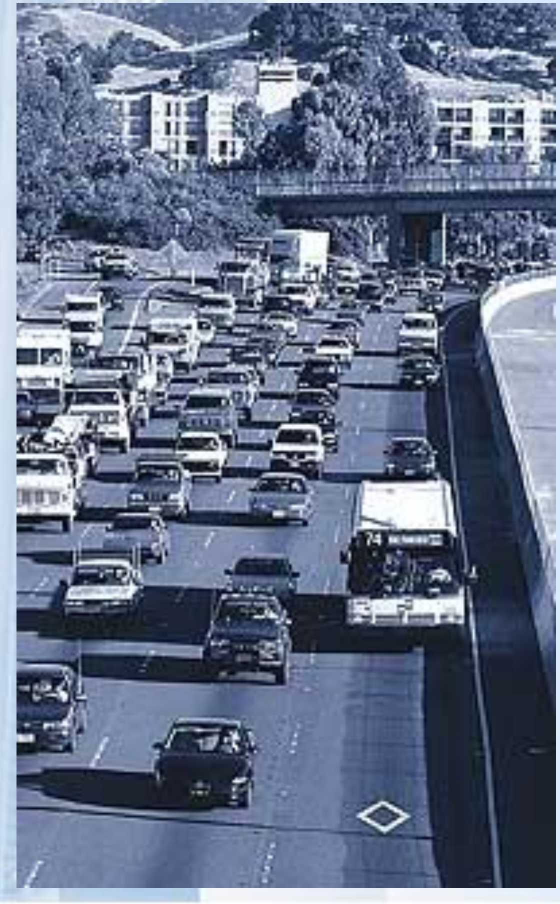
Source: http://www.theseattletraveler.com/

HOV Lane: the Solution?

Empty HOV Lane, Southbound US-59 Houston, Texas

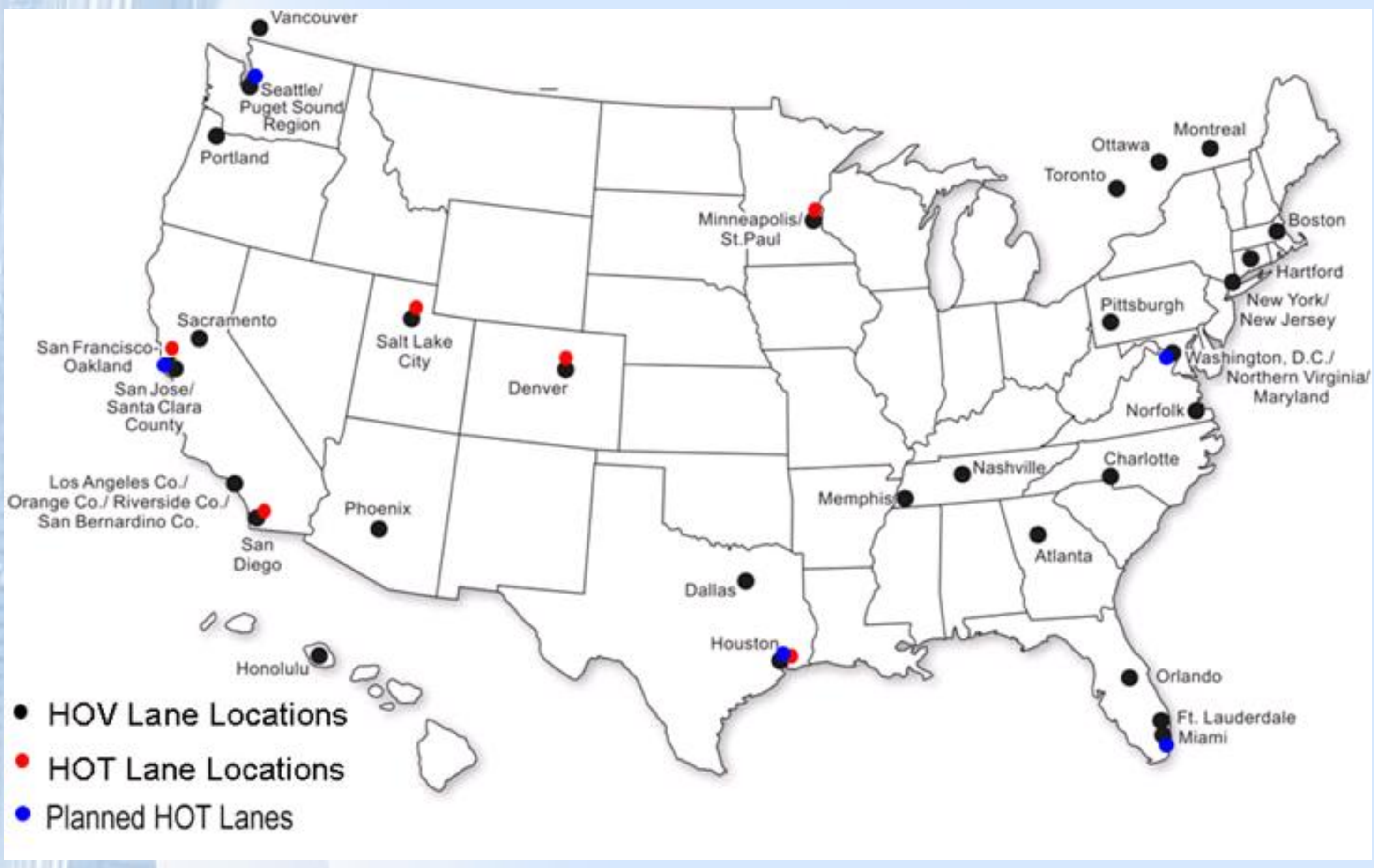
Source: http://flickr.com/photos/46511298@N00/785824829/

HOV or Toll = High Occupancy Toll (HOT)



Sources: http://www.mtc.ca.gov/news/transactions/ta11-1204/vision.htm and http://www.vta.org/projects/hot_lanes/hot_final.pdf

Existing HOT Lane Systems



Source:http://ops.fhwa.dot.gov/speeches/ntoc2007/index.htm

HOT Lane Pricing Strategies

- Fixed toll rate
- Varying toll rate by time of day
- Dynamic toll rate

Facility	Project Length	Minimum Toll	Maximum Toll or Highest Toll Charged
I-394	9 miles	$0.25 per section ($0.50 for full-length trip)	$8.00
I-15	8 miles	$0.50[1]	$8.00[2]
91 Express Lanes	10 miles	$1.15 during overnight hours; $1.85 during midday hours	$6.65 to $9.50 during evening peak hours, depending on day of week
I-25 Express Lanes	5 to 7 miles, depending on access point	$0.50	$3.25

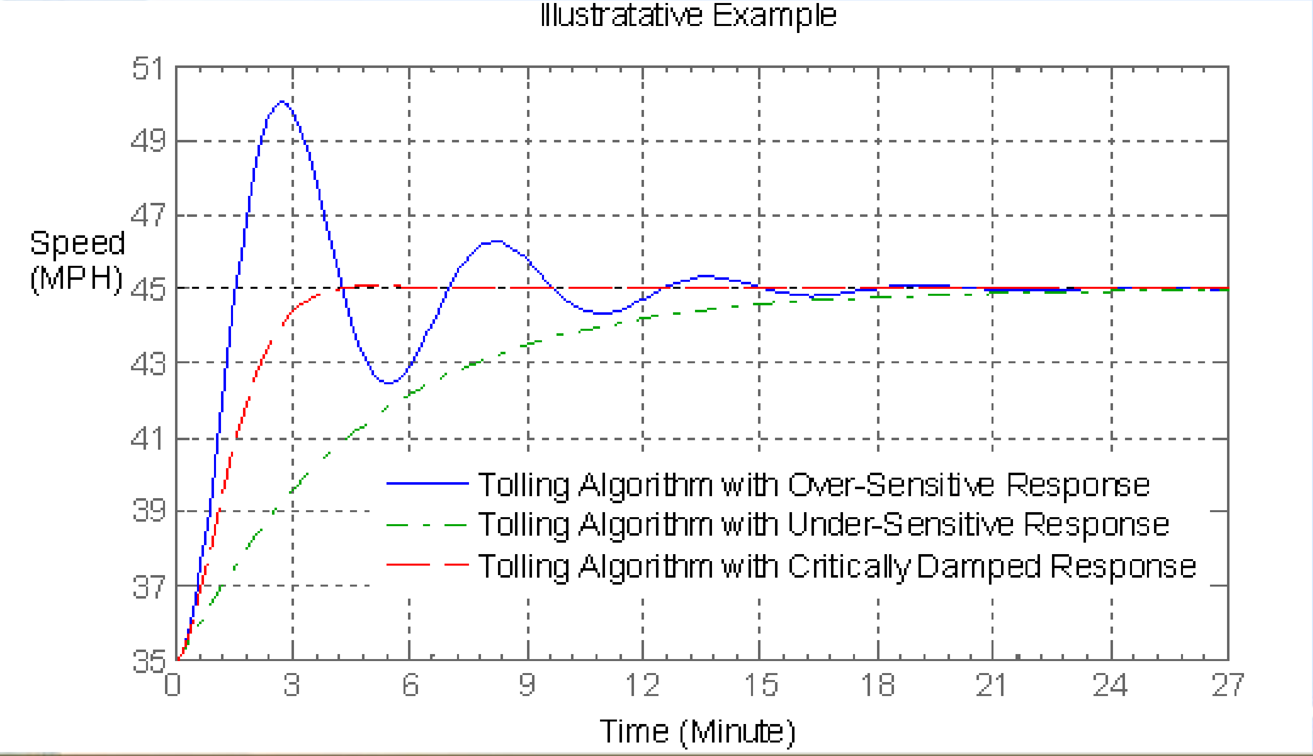
[1] Consideration is being given to increasing the minimum to $1.25 to enhance revenues.
[2] The published maximum toll in charts on Sandag's website is $4, although rates can go as high as $8 when needed to meet the project's speed objective.

Time Period	Toll Rates to Maximize Usage(1)					
	New Base(2)		Higher VOT(3)		Higher Traffic (4)	
	Northbound	Southbound	Northbound	Southbound	Northbound	Southbound
2008						
AM Peak 1 (5:30-7:00 a.m.)	$1.00	$0.50	$2.00	$0.50	$1.75	$0.50
AM Peak 2 (7:00-8:00 a.m.)	0.75	0.50	2.00	0.50	2.00	0.50
AM Shoulder (8:00-9:00 a.m.)	0.50	0.50	0.50	0.50	0.50	0.50
Midday (9:00 a.m.-2:30 p.m.)	0.50	0.50	0.50	0.50	0.50	0.50
PM Shoulder 1 (2:30-4:00 p.m.)	0.50	0.75	0.50	1.75	0.50	2.25
PM Peak 1 (4:00-5:00 p.m.)	0.50	2.25	0.50	**5.00**	0.50	**7.00**
PM Peak 2 (5:00-6:00 p.m.)	0.50	1.75	0.50	4.00	0.50	**5.50**
PM Shoulder 2 (6:00-7:00 p.m.)	0.50	0.50	0.50	1.00	0.50	1.50

Source: www.wstc.wa.gov/AgendasMinutes/

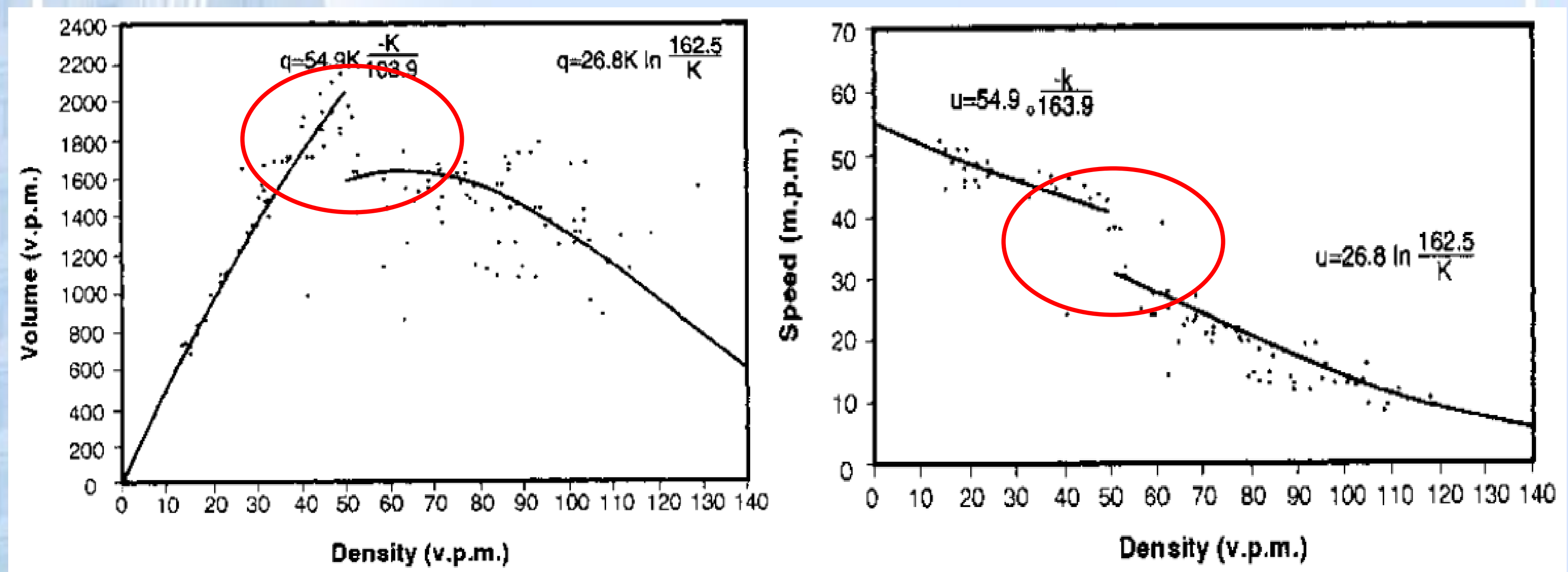
Major Problems with Current Tolling Strategies

- Major Problems
  - Under-sensitive tolling strategy
  - Over-sensitive tolling strategy



Major Problems with Current Tolling Strategies

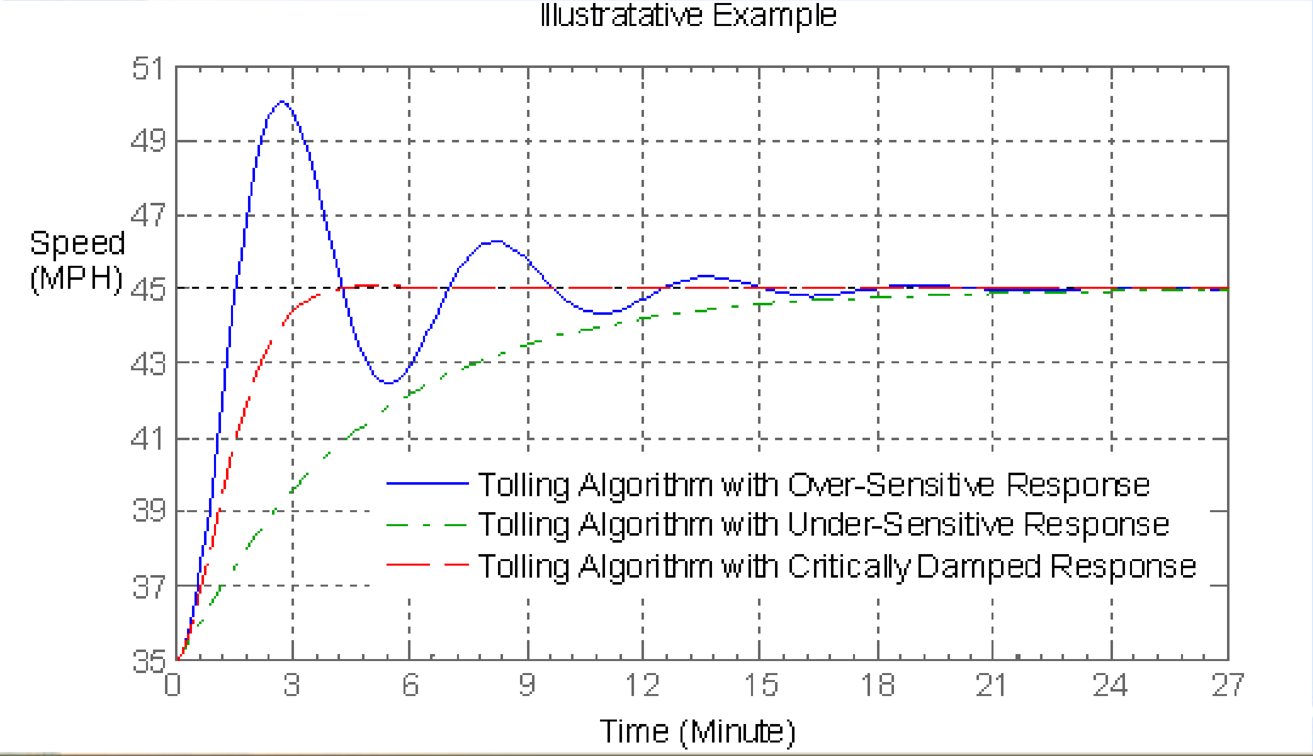
- Reliability vs. Degree of HOT lane utilization



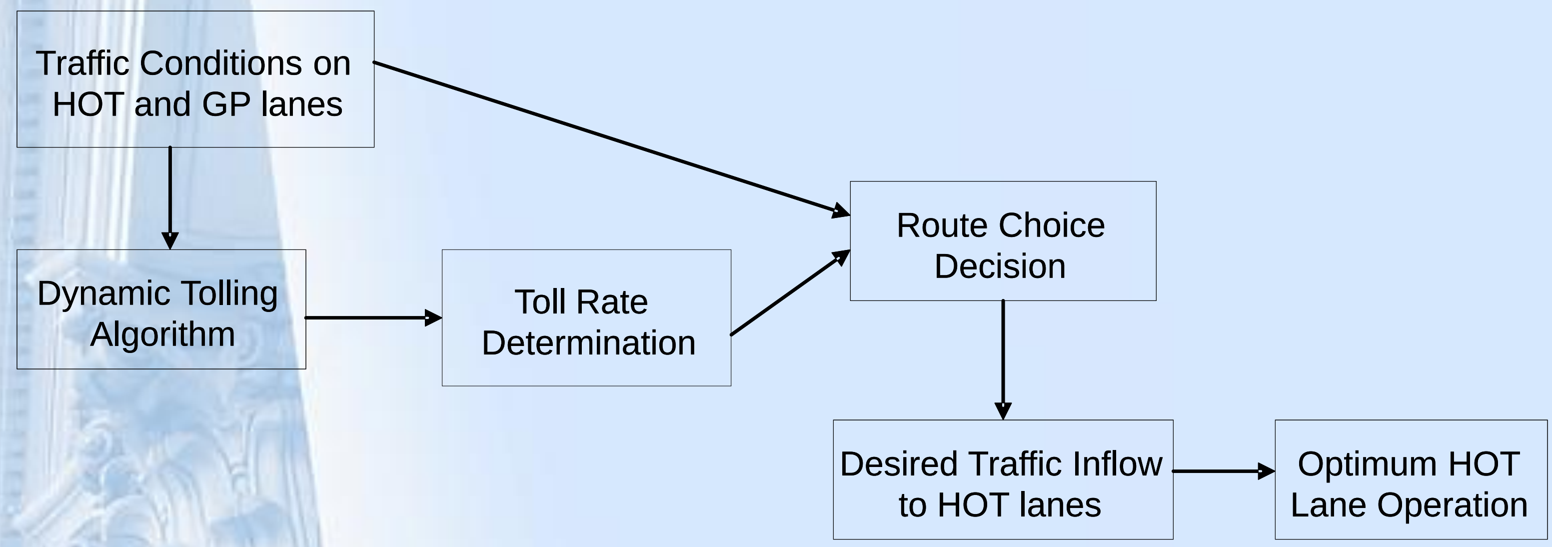
Edie's Hypothesis Fitted to Chicago Data (Drake et al., 1967)

Major Problems with Current Tolling Strategies

- Major Problems
  - Under-sensitive tolling strategy
  - Over-sensitive tolling strategy



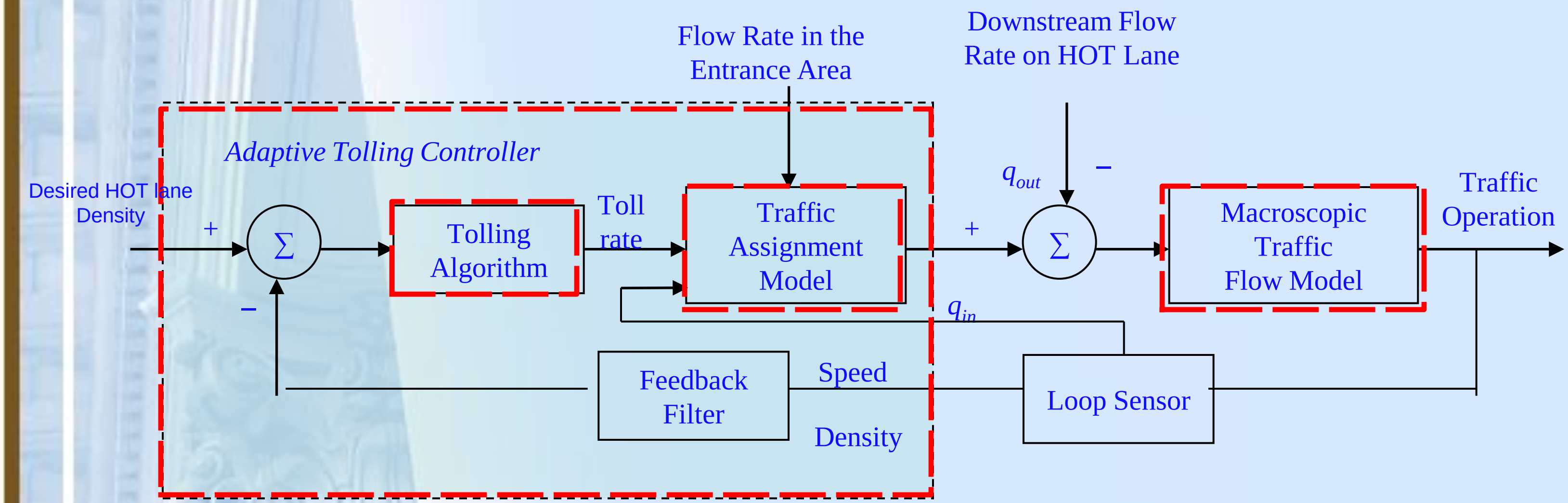
HOT Lane System Operation Mechanism



Representative HOT Lane System Operation Mechanism

Toll ↔ HOT lane traffic flow

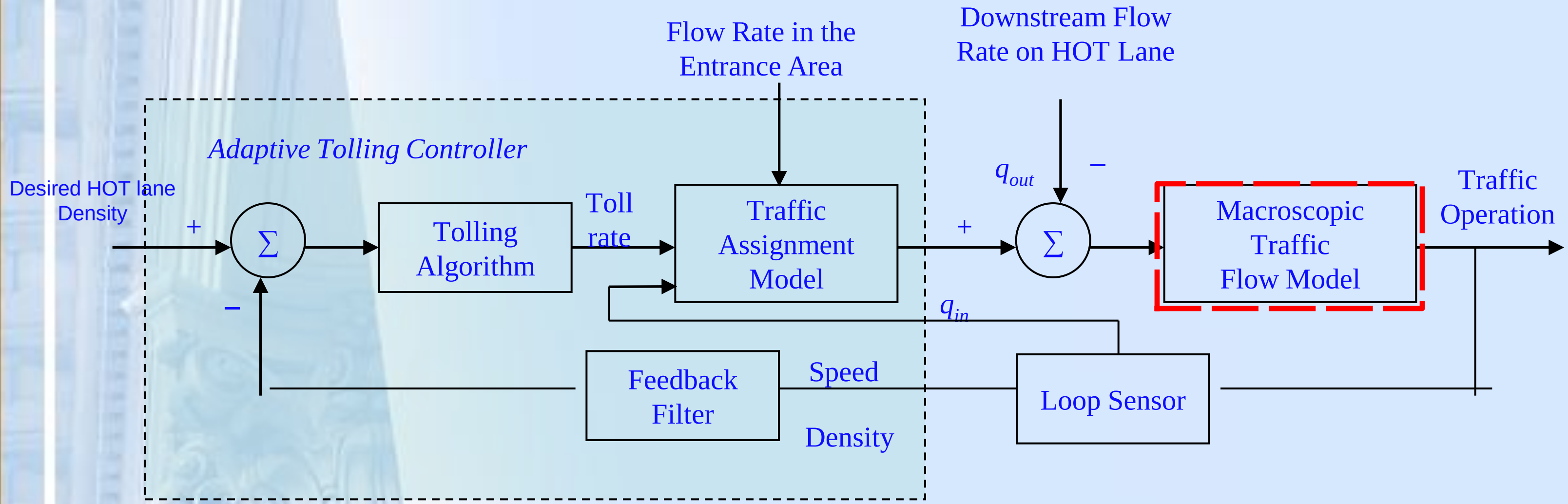
Self-Adaptive Tolling System Architecture



The Schematic Architecture of the Proposed Self-Adaptive Tolling System

Desired HOT lane traffic flow ⟷ Appropriate toll rate

Methodology



The Schematic Architecture of the Proposed Self-Adaptive Tolling System

Methodology

- Model traffic evolution on HOT lanes

The **LWR** (Lighthill-Whitham-Richards)traffic flow model

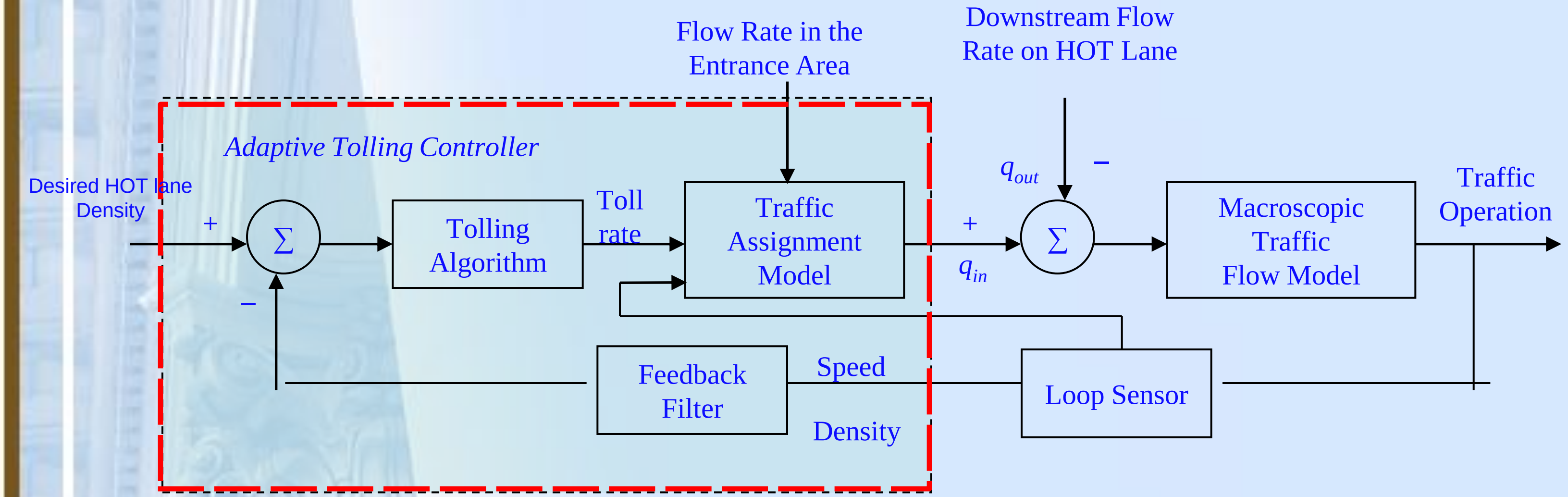
$$\frac{\partial \rho(x,t)}{\partial t} + \frac{\partial q(x,t)}{\partial x} = 0$$

Assume the HOT lane starts at the point x_1 and ends at the point x_2 along a freeway corridor, and traffic is distributed uniformly along the HOT lane

$$\frac{d}{dt}\int_{x_1}^{x_2} \rho(x,t)dx + q(x_2,t) - q(x_1,t) = 0$$

$$\frac{d\rho^*(t)}{dt} = \frac{q(x_1,t) - q(x_2,t)}{x_2 - x_1}$$

Methodology



The Schematic Architecture of the Proposed Self-Adaptive Tolling System

Methodology

- Tolling controller construction

Laplace transform: Convert a system between time-domain and frequency-domain

$$H(s)=\int_{0^-}^{\infty} h(t)\exp(-st)dt$$

The HOT system transfer function can be calculated as follows based on the convolution theorem

$$G_t(s)=\frac{L\{\rho(t)\}}{L\{q(x_1,t)-q(x_2,t)\}}=\frac{L\{\rho(t)\}}{L\{\frac{d\rho^*(t)}{dt}(x_2-x_1)\}}$$

$$=\frac{\int_{0^-}^{\infty}\rho(t)\exp(-st)dt}{(x_2-x_1)*\int_{0^-}^{\infty}\frac{d\rho^*(t)}{dt}\exp(-st)dt}=\frac{\rho(s)}{(x_2-x_1)*(s\rho(s)-\rho(0^-))}=\frac{1}{s*(x_2-x_1)}$$

Methodology

- Tolling controller construction

Design the Proportional (P) controller and Integral (I) controller components

$$D(s) = k_P + \frac{k_L}{s}$$

The transfer function of the overall HOT lane system

$$G(s) = \frac{D(s) * G_t(s)}{1 + D(s) * G_t(s)} = \frac{(k_P + \frac{k_L}{s}) * \frac{1}{s * (x_2 - x_1)}}{1 + (k_P + \frac{k_L}{s}) * \frac{1}{s * (x_2 - x_1)}} = \frac{k_P s + k_L}{(x_2 - x_1)s^2 + k_P s + k_L}$$

Methodology

- Tolling controller construction

Re-organize the HOT lane system transfer function

$$G(s)=\frac{k_P s+k_L}{Ms^2+k_P s+k_L}=\frac{(k_P/M)s+k_L/M}{s^2+(k_P/M)s+k_L/M}=\frac{2\zeta\omega_n s+\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$$

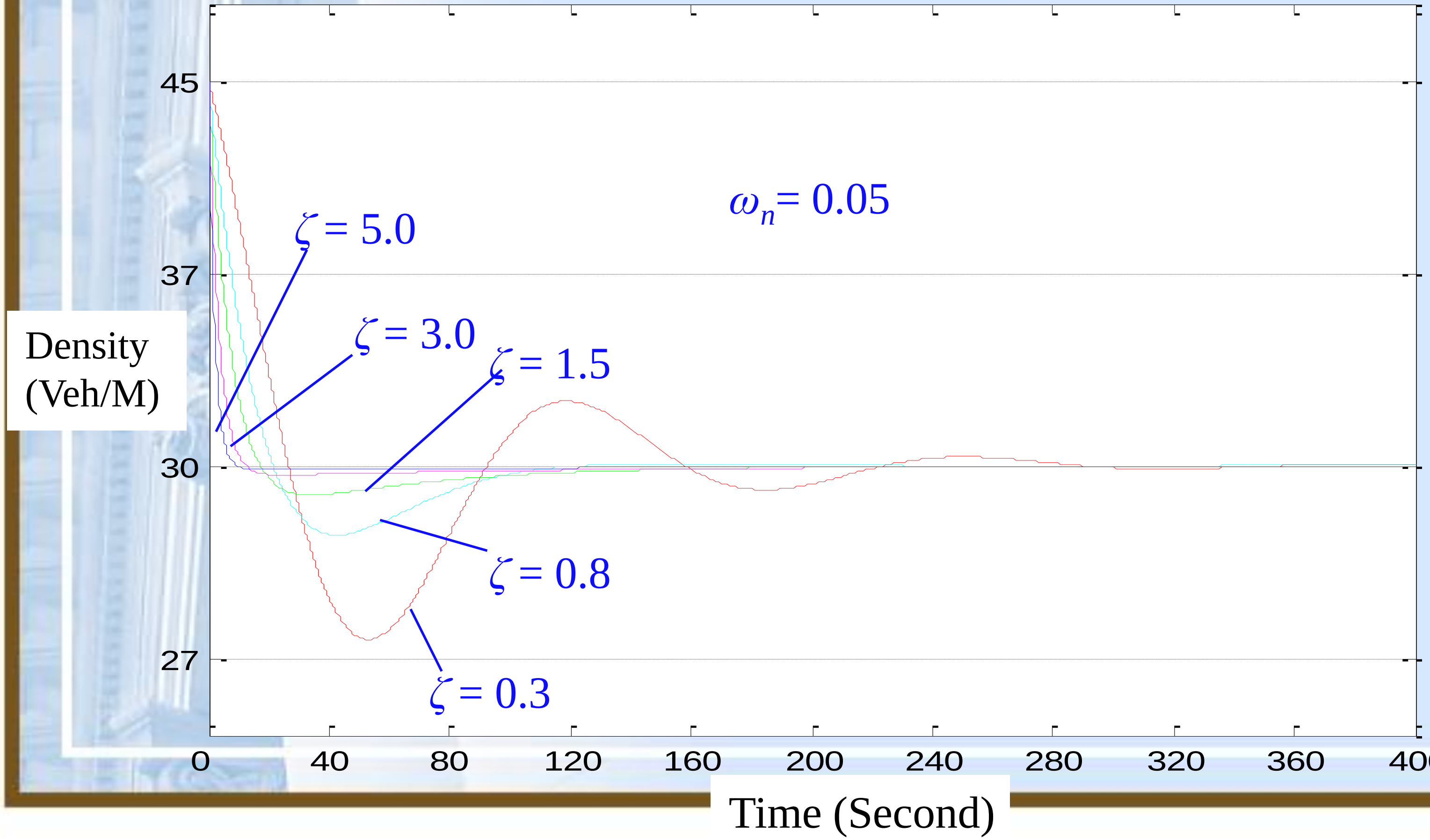
$$g(t)=L^{-1}\{G(s)\}=\frac{1}{2\pi j}\int_{\sigma_c-j\infty}^{\sigma_c+j\infty}G(s)\exp(st)ds$$

$$=\begin{cases}\exp(-\zeta\omega_n t)*(2\zeta\omega_n\cos(\omega_n\sqrt{1-\zeta^2}t)+\dfrac{\omega_n(1-2\zeta^2)}{\sqrt{1-\zeta^2}}\sin(\omega_n\sqrt{1-\zeta^2}t)) & 0\le\zeta\le 1\\ (\zeta\omega_n+\dfrac{\omega_n(2\zeta^2-1)}{2\sqrt{\zeta^2-1}})\exp(-(\zeta\omega_n+\omega_n\sqrt{\zeta^2-1})t)+(\zeta\omega_n-\dfrac{\omega_n(2\zeta^2-1)}{2\sqrt{\zeta^2-1}})\exp(-(\zeta\omega_n-\omega_n\sqrt{\zeta^2-1})t) & 1<\zeta\end{cases}$$

Methodology

- Tolling controller construction

  The system responses



Methodology

- Tolling controller construction

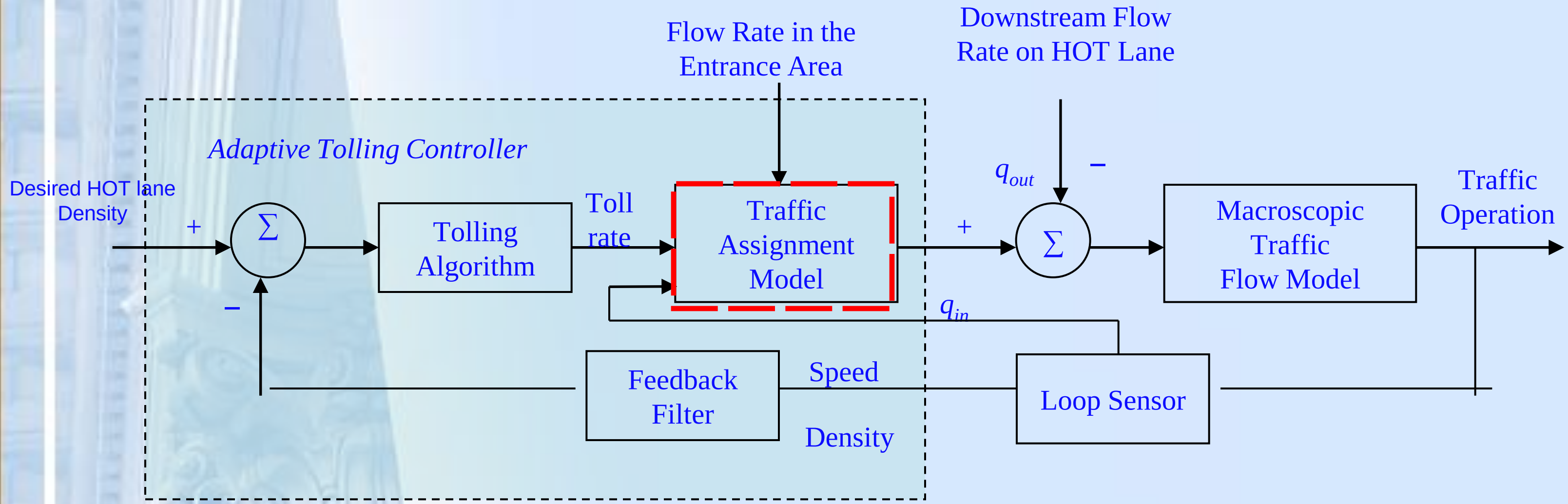
Digitize the tolling controller

$$D(z) = D(s)\Big|_{s=\frac{2}{T_s}*\frac{z-1}{z+1}} = k_P + \frac{k_L}{s}\Big|_{s=\frac{2}{T_s}*\frac{z-1}{z+1}} = \frac{A_1 z + A_2}{A_3 z + A_4}$$

The desired flow rates to HOT lanes can be computed

$$F_{HOT}(t) = \frac{\rho_r*(A_1 + A_2) - (A_1\rho*(t) + A_2\rho*(t-1)) - A_4 F_{HOT}(t-1)}{A_3}$$

Methodology



The Schematic Architecture of the Proposed Self-Adaptive Tolling System

Methodology

- Modeling traffic assignment using discrete choice model, Logit model

$$TC_{HOT} = \alpha * TT_{HOT} + TR_{HOT} \qquad TC_{GP} = \alpha * TT_{GP} + TR_{GP} = \alpha * TT_{GP}$$

$$U_{HOT} = \frac{1}{TC_{HOT}} = \frac{1}{\alpha * TT_{HOT} + TR_{HOT}}$$

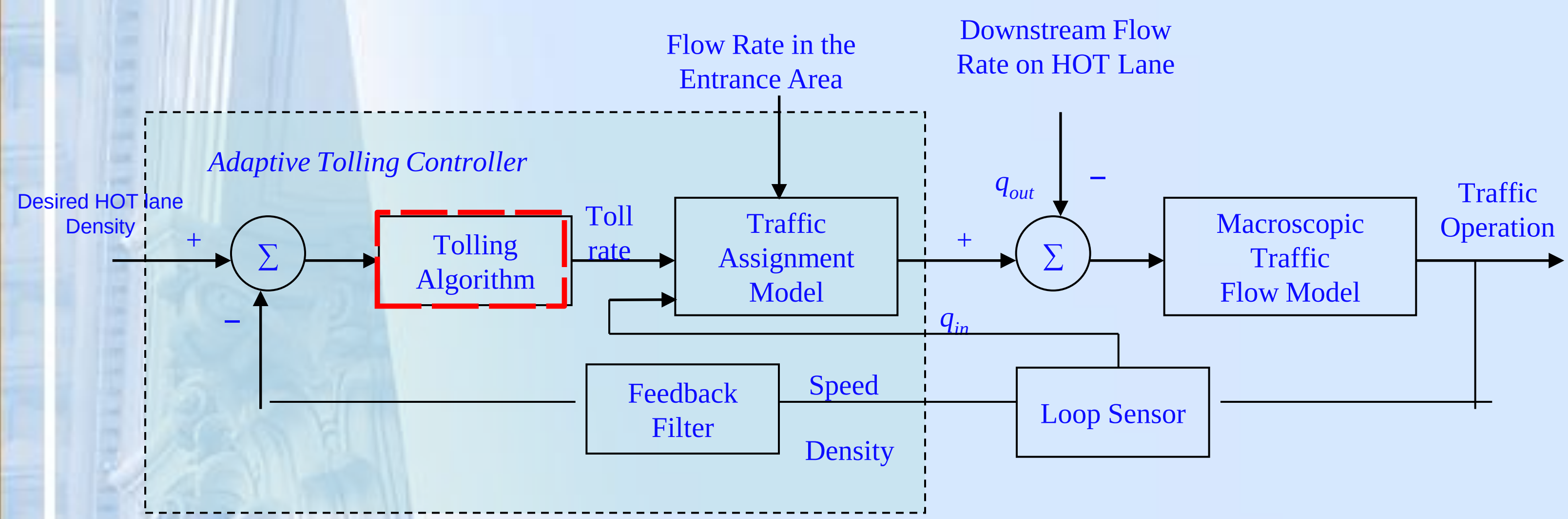
$$U_{GP} = \frac{1}{TC_{GP}} = \frac{1}{\alpha * TT_{GP}}$$

$$F_{HOT} = F_{total} * P_{HOT} = F_{total} * \frac{\exp(U_{HOT})}{\exp(U_{HOT}) + \exp(U_{GP})}$$

$$= F_{total} * f(TR_{HOT}, TT_{HOT}, TT_{GP})$$

Methodology

- Toll Rate Estimation



The Schematic Architecture of the Proposed Self-Adaptive Tolling System

$$TR_{HOT} = f^{-1}(F_{HOT}/F_{total}, TT_{HOT}, TT_{GP}) = \frac{1}{\frac{1}{\alpha \cdot TT_{GP}} - \ln\left(\frac{1 - F_{HOT}/F_{total}}{F_{HOT}/F_{total}}\right)} - \alpha \cdot TT_{HOT}$$

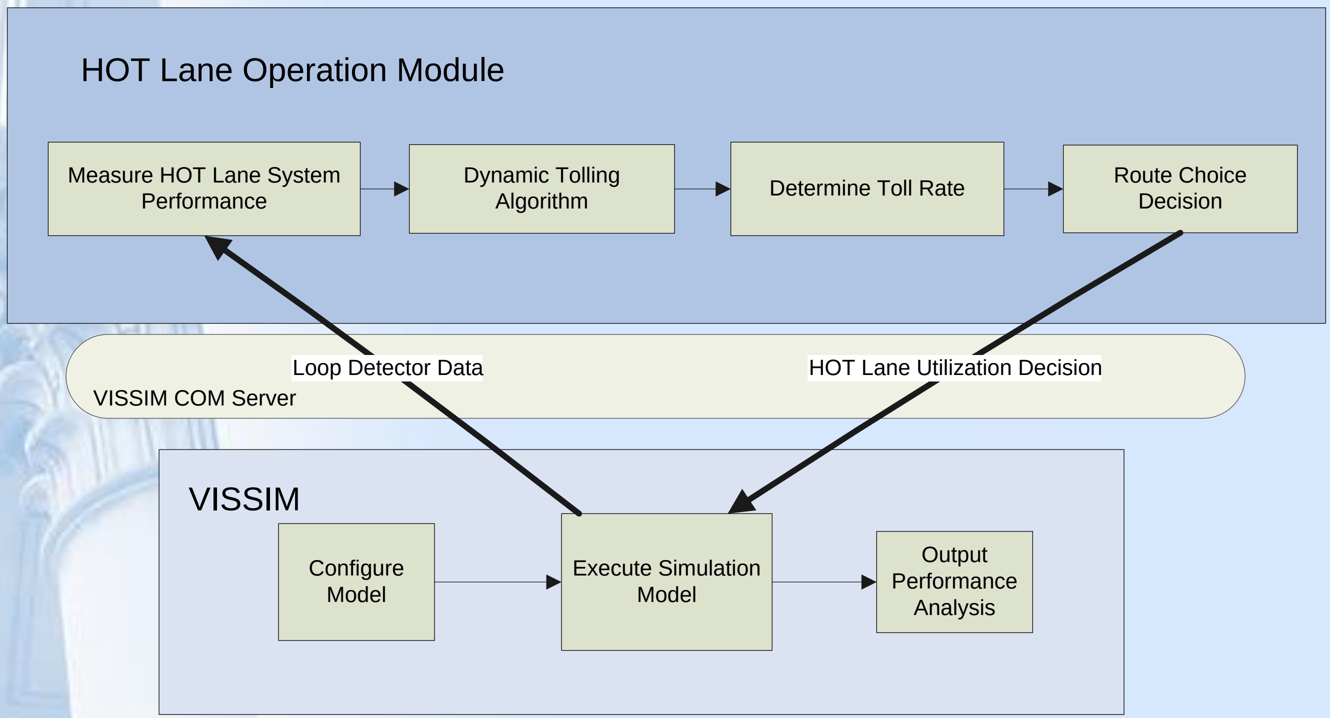
Simulation Experiments

- VISSIM Simulation Model Development
  - ✓ VISSIM's limitations

Source: www.ptvamerica.com/vissim.html

Simulation Experiments

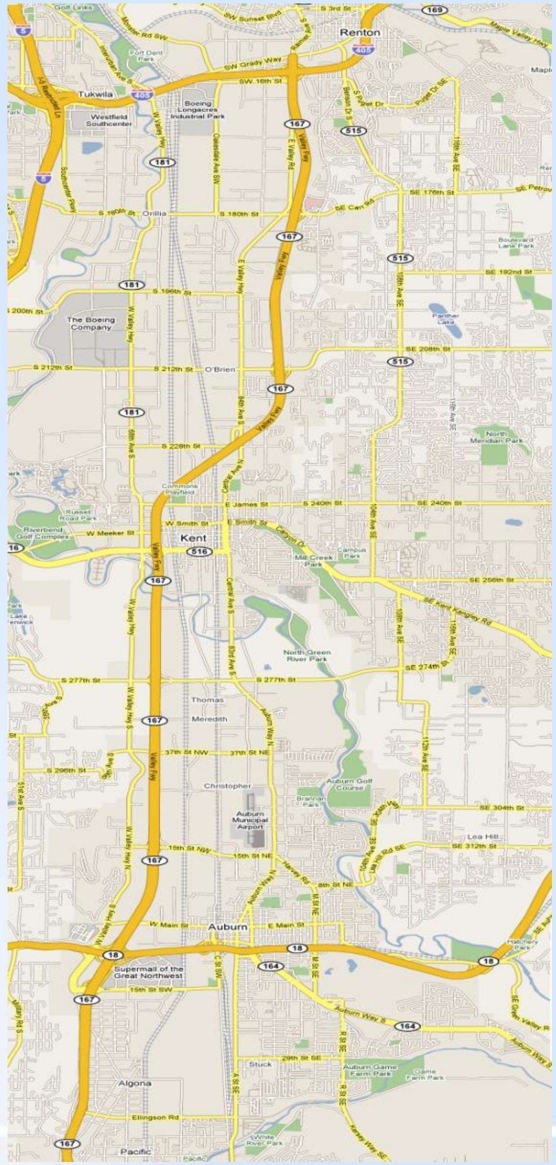
- VISSIM Simulation Model Development
  - ✓ External HOT lane control Module Development

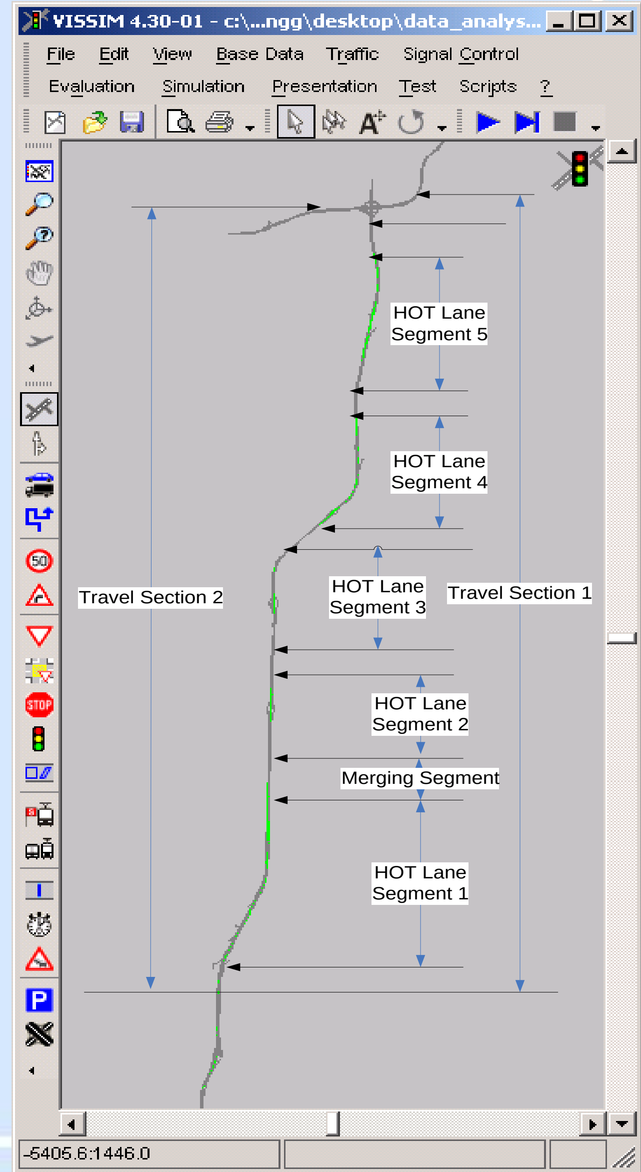


HOT Lane Simulation Model

Simulation Model Configuration for Washington SR-167

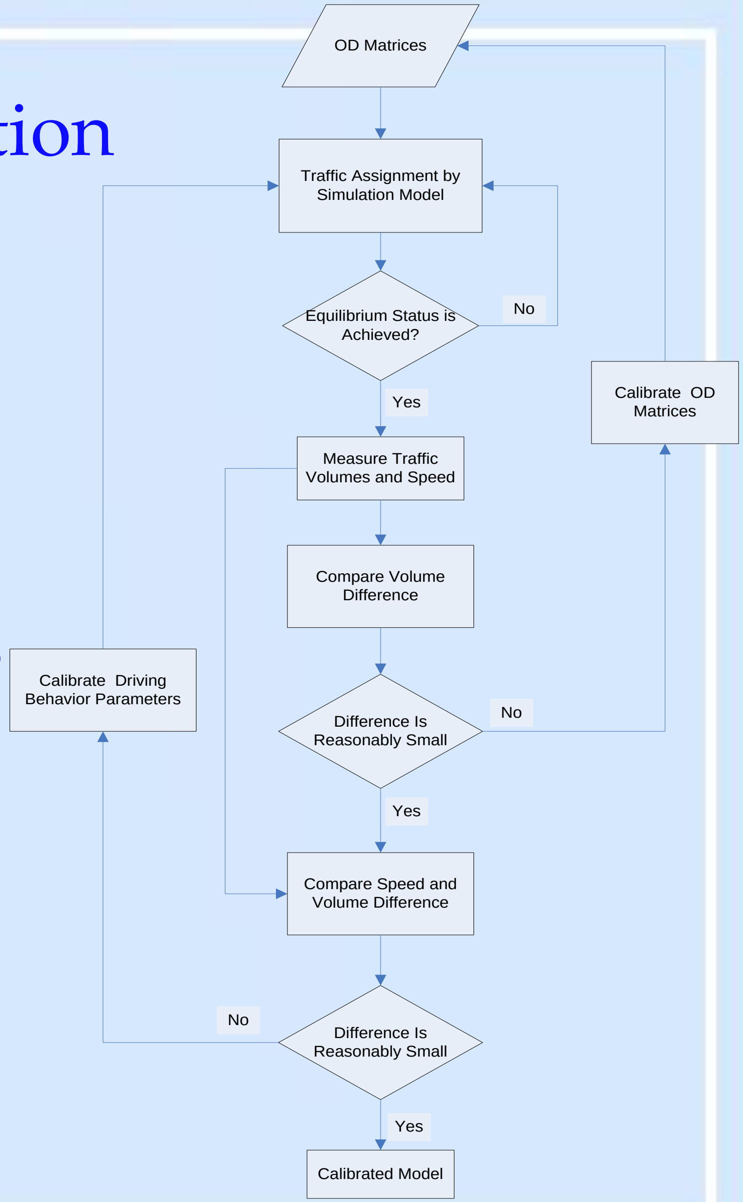
- ✓**Modeling Scope**
- ✓**Simulation Time Period**
  6:00 to 9:00am
- ✓**Traffic Composition**
  SOVs, HOVs and Trucks





Simulation Model Calibration

- Dynamic Traffic Assignment
- HOV lane Implementation
- Simulation Model Calibration
  - Data Source
    - *Traffic Management Center Summary Report: SR-167 Ramp and Roadway 2006 Traffic Volumes.*
    - *SR-167 and I-405 Annual Average Traffic Volumes and Speeds in 2005.*



Flow Chart of the Calibration Procedure

Simulation Results and Discussions

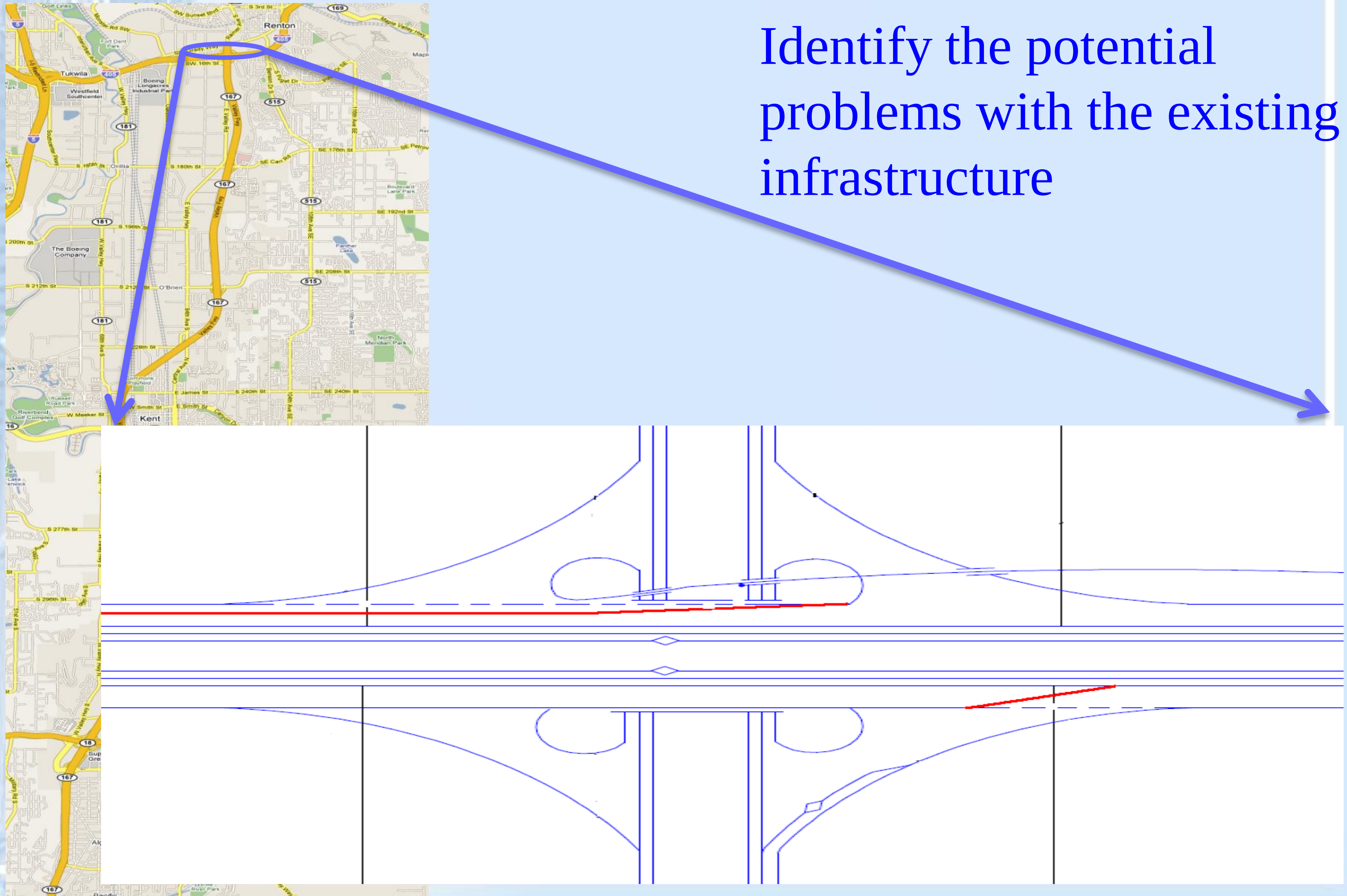
- Current Traffic Demands

Simulation Time Period: 6:00-9:00 Am		HOV Operation			HOT Operation			Improvement	
		TT	TP	SP	TT	TP	SP	TP	SP
Merging Area 1		57.9	7747	34.2	64.3	7732	30.8	-0.2%	-9.9%
HOT Segment 1: Length = 2.8 M	GP+HOV/HOT	230.0	5588	43.6	176.1	5586	56.9	0.0%	30.6%
	GP Lane	242.0	4677	41.4	180.8	2573	55.4	-45.0%	33.8%
	HOV/HOT Lane	168.4	912	59.5	172.1	3013	58.3	230.5%	-2.1%
	On-Ramps	71.7	5401	24.5	32.7	5445	52.2	0.8%	113.0%
	Off-Ramps	22.5	2712	34.2	20.7	2684	36.9	-1.0%	7.9%
Merging Area 2		44.6	10142	48.7	44.6	10210	48.7	0.7%	0.0%
HOT Segment 2: Length = 1.3 M	GP+HOV/HOT	107.3	9153	43.6	85.0	9226	55.0	0.8%	26.2%
	GP Lane	113.3	7579	41.2	87.3	6796	53.5	-10.3%	29.8%
	HOV/HOT Lane	77.9	1574	60.0	78.9	2430	59.2	54.4%	-1.2%
	On-Ramps	42.8	1588	43.6	38.7	1592	48.0	0.2%	10.0%
	Off-Ramps	43.6	893	48.6	39.0	899	53.1	0.7%	9.3%

Integrated Simulation Results under Existing Traffic Demands From 7 Simulation Runs

Simulation Results and Discussions

Identify the potential problems with the existing infrastructure



Simulation Results and Discussions

Simulation Time Period: 6:00-9:00 Am		HOV Operation			HOT Operation			Improvement	
		TT	TP	SP	TT	TP	SP	TP	SP
	GP+HOV/HOT	80.7	7976	57.9	80.5	7945	58.1	-0.4%	0.2%
	GP Lane	81.4	6384	57.4	81.2	6287	57.6	-1.5%	0.2%
	HOV/HOT Lane	77.8	1592	60.0	77.7	1659	60.1	4.2%	0.1%
	On-Ramps	35.7	1321	51.9	35.5	1321	52.1	0.1%	0.5%
80%	Off-Ramps	38.5	776	53.9	38.4	774	53.9	-0.3%	0.0%
	GP+HOV/HOT	83.2	8576	56.1	81.7	8560	57.2	-0.2%	1.9%
	GP Lane	84.4	6985	55.3	82.8	6568	56.4	-6.0%	1.9%
	HOV/HOT Lane	77.9	1591	60.0	78.0	1992	59.9	25.2%	-0.2%
	On-Ramps	37.6	1443	49.3	36.7	1444	50.5	0.0%	2.5%
90%	Off-Ramps	38.9	833	53.3	38.8	832	53.5	-0.1%	0.3%
	GP+HOV/HOT	107.3	9153	43.6	85.0	9226	55.0	0.8%	26.2%
	GP Lane	113.3	7579	41.2	87.3	6796	53.5	-10.3%	29.8%
	HOV/HOT Lane	77.9	1574	60.0	78.9	2430	59.2	54.4%	-1.2%
	On-Ramps	42.8	1588	43.6	38.7	1592	48.0	0.2%	10.0%
100%	Off-Ramps	43.6	893	48.7	39.0	899	53.2	0.7%	9.2%
	GP+HOV/HOT	134.6	9117	34.7	93.4	9489	50.0	4.1%	44.0%
	GP Lane	145.7	7621	32.1	98.7	6547	47.3	-14.1%	47.7%
	HOV/HOT Lane	78.1	1496	59.8	81.8	2942	57.1	96.7%	-4.5%
	On-Ramps	47.5	1691	39.1	41.8	1700	44.3	0.5%	13.3%
110%	Off-Ramps	53.4	889	39.1	39.3	913	52.8	2.6%	35.0%
	GP+HOV/HOT	183.1	8764	25.5	117.6	9279	39.7	5.9%	55.8%
	GP Lane	202.2	7410	23.1	127.8	6643	36.5	-10.4%	58.2%
	HOV/HOT Lane	78.4	1354	59.6	88.5	2636	52.8	94.7%	-11.4%
	On-Ramps	55.0	1819	33.8	46.8	1812	40.4	-0.4%	19.5%
120%	Off-Ramps	75.3	897	27.6	46.6	924	45.4	3.0%	64.9%
	GP+HOV/HOT	211.5	8330	22.1	131.0	9311	35.7	11.8%	61.5%
	GP Lane	234.8	7087	19.9	146.0	6350	32.0	-10.4%	60.8%
	HOV/HOT Lane	78.9	1243	59.2	96.3	2961	48.5	138.2%	-18.0%
	On-Ramps	75.4	1949	24.9	57.1	1947	34.4	-0.1%	38.5%
130%	Off-Ramps	90.3	893	23.0	52.3	942	41.2	5.6%	79.2%
	GP+HOV/HOT	255.2	7859	18.3	157.6	8277	29.6	5.3%	62.0%
	GP Lane	284.3	6747	16.4	176.8	6093	26.4	-9.7%	60.8%
	HOV/HOT Lane	79.1	1112	59.1	99.2	2184	47.1	96.3%	-20.3%
	On-Ramps	103.0	2073	18.2	59.2	1910	31.5	-7.8%	73.3%
140%	Off-Ramps	112.2	846	18.5	60.8	857	35.5	1.3%	92.1%

Integrated Simulation Results for HOT lane Segment 2 under Various Traffic Demands From 7 Simulation Runs

Conclusions

- Optimizing HOT lane operations can generate significant economic returns and social benefits.
- A new self-adaptive tolling algorithm is developed to dynamically optimize HOT lane operations.
- VISSIM-based simulation tests show this tolling algorithm performs reasonably well under various test scenarios.
- Next Step...
  - Implementation
  - Further tests

Thanks for your attention!

Question?